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HIGHER-ORDER CORRECTIONS TO RADIATIVE Υ DECAYS^a

MICHAEL KRÄMER
*Theoretical Physics Division, CERN,
1211 Geneva 23, Switzerland
E-mail: Michael.Kraemer@cern.ch*

Recent advances in the theoretical description of radiative Υ decays are reviewed, including the calculation of next-to-leading order QCD corrections to the photon spectrum.

1 Introduction

The calculations of heavy quarkonium decay rates are among the earliest applications of perturbative QCD¹ and have been used to extract information on the QCD coupling at scales of the order of the heavy-quark mass. A consistent and rigorous framework for treating inclusive quarkonium decays has recently been developed, superseding the earlier non-relativistic potential model. The factorization approach is based on the use of non-relativistic QCD (NRQCD) to separate the short-distance physics of heavy-quark annihilation from the long-distance physics of bound-state dynamics.² The annihilation decay rate can be expressed as a sum of terms, each of which factors into a short-distance coefficient and a long-distance matrix element:

$$\Gamma(H \rightarrow X) = \sum_n C(Q\bar{Q}[n] \rightarrow X) \langle H | \mathcal{O}_n | H \rangle. \quad (1)$$

The sum extends over all possible local 4-fermion operators \mathcal{O}_n in the effective Lagrangian of NRQCD that annihilate and create a heavy-quark–antiquark pair. The short-distance coefficients C are proportional to the rates for an on-shell $Q\bar{Q}$ pair in a colour, spin and angular-momentum configuration n to annihilate into a given final state X , and can be calculated perturbatively in the strong coupling $\alpha_s(m_Q)$. The long-distance factors on the other hand are

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well-defined matrix elements that can be evaluated numerically using lattice simulations of NRQCD.³ For small average velocities v of the heavy quark in the quarkonium rest frame, each of the NRQCD matrix elements scales with a definite power of v and the general expression of Eq. (1) can be organized into an expansion in powers of the heavy-quark velocity.

At leading order in the velocity expansion, the NRQCD description of S -wave quarkonium decays is equivalent to the non-relativistic potential model, where the non-perturbative dynamics is described by a single long-distance factor related to the bound state's wave function at the origin.^b The NRQCD approach improves upon the potential model calculations for S -wave quarkonia by providing a rigorous non-perturbative definition of the long-distance factor, so this can be calculated using lattice simulations, and by allowing the systematic inclusion of relativistic corrections due to the motion of the heavy quarks in the bound state.

Using the NRQCD factorization approach it is, in principle, possible to calculate the annihilation decay rates of heavy quarkonium from first principles, the only inputs being the heavy-quark mass m_Q and the QCD coupling $\alpha_s(m_Q)$. Fixing the heavy-quark mass from, for example, sum rule calculations for quarkonia, the analysis of quarkonium decay rates can provide useful information on the QCD coupling at the heavy-quark mass scale.

For a reliable extraction of $\alpha_s(m_Q)$, however, the theoretical uncertainties in the calculation of quarkonium decay rates have to be analysed carefully, and the effects of higher-order corrections in both the perturbative expansion and the velocity expansion have to be included. Radiative Υ decays are particularly well suited to study the impact of higher-order corrections, since not only the total decay rate but also the photon spectrum can be measured and compared with theory.

2 The leading-order decay rate $\Upsilon \rightarrow \gamma + X$

At leading order in the velocity expansion, radiative Υ decays proceed through the annihilation of a colour-singlet $n = {}^3S_1^{(1)}$ $b\bar{b}$ pair. (We use spectroscopic notation with the superscript in brackets denoting the colour state.) The direct contribution, where the photon is radiated off a heavy quark, is shown in Fig. 1(a). The corresponding leading-order decay rate is given by

^bIn the case of P -wave quarkonia, the potential model calculations at $\mathcal{O}(\alpha_s^3)$ encounter infrared divergences, which can not be factored into a single non-perturbative quantity. Within the NRQCD approach, this problem finds its natural solution since the infrared singularities are factored into a long-distance matrix element related to higher Fock-state components of the quarkonium wave function.

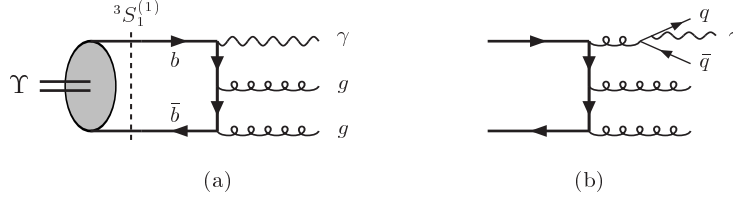


Figure 1: Generic leading order Feynman diagrams for radiative Υ decay: (a) direct contribution, (b) fragmentation contribution.

$$\Gamma_{\text{dir}}^{\text{LO}}(\Upsilon \rightarrow \gamma X) = \frac{16}{27} \alpha \alpha_s^2 e_b^2 (\pi^2 - 9) \frac{\langle \Upsilon | \mathcal{O}_1(^3S_1) | \Upsilon \rangle}{m_b^2}, \quad (2)$$

where e_b is the charge of the b quark.⁴ Up to corrections of $\mathcal{O}(v^4)$, the non-perturbative NRQCD matrix element is related to the Υ wave function at the origin through $\langle \Upsilon | \mathcal{O}_1(^3S_1) | \Upsilon \rangle \approx 2N_C |\varphi(0)|^2$ with $N_C = 3$ the number of colours. More information about the decay dynamics is provided by the shape of the photon energy spectrum, which, at leading order in the velocity expansion and for the direct term, can be predicted within perturbation theory

$$\frac{1}{\Gamma_{\text{dir}}^{\text{LO}}} \frac{d\Gamma_{\text{dir}}^{\text{LO}}(\Upsilon \rightarrow \gamma X)}{dx_\gamma} = \frac{2}{(\pi^2 - 9)} \left(2 \frac{1 - x_\gamma}{x_\gamma^2} \ln(1 - x_\gamma) - 2 \frac{(1 - x_\gamma)^2}{(2 - x_\gamma)^3} \ln(1 - x_\gamma) + \frac{2 - x_\gamma}{x_\gamma} + x_\gamma \frac{1 - x_\gamma}{(2 - x_\gamma)^2} \right), \quad (3)$$

where $0 \leq x_\gamma = E_\gamma/m_b \leq 1$.⁵ To very good accuracy, Eq. (3) can be approximated by a linear spectrum: $1/\Gamma_{\text{dir}}^{\text{LO}} d\Gamma_{\text{dir}}^{\text{LO}}/dx_\gamma \approx 2x_\gamma$.

It has been pointed out recently that there is an additional leading-order contribution to radiative Υ decays, where the photon is produced by fragmentation, i.e. by collinear emission from light quarks, see Fig. 1(b).⁶ The fragmentation contribution, although of $\mathcal{O}(\alpha\alpha_s^4)$ in the coupling constant, is enhanced by a double logarithmic singularity $\sim \ln^2(m_b^2/\Lambda^2) \sim 1/\alpha_s^2$ arising from the phase-space region where both the light-quark-antiquark splitting as well as the photon emission become collinear. The complete LO photon spectrum is thus given by

$$\frac{d\Gamma^{\text{LO}}(\Upsilon \rightarrow \gamma X)}{dx_\gamma} = \frac{d\Gamma_{\text{dir}}^{\text{LO}}}{dx_\gamma} + \int_{x_\gamma}^1 \frac{dz}{z} C_g(z) D_g^\gamma\left(\frac{x_\gamma}{z}\right), \quad (4)$$

where the first term on the right-hand side denotes the direct cross section,

Eq. (3), while the second term represents the contribution from gluon fragmentation into a photon.⁶ The fragmentation function D_g^γ is sensitive to non-perturbative effects and has to be extracted from experiment. The relative size of the direct component and the fragmentation contribution to the photon spectrum has been studied at leading order and is shown in Fig. 2. The LO

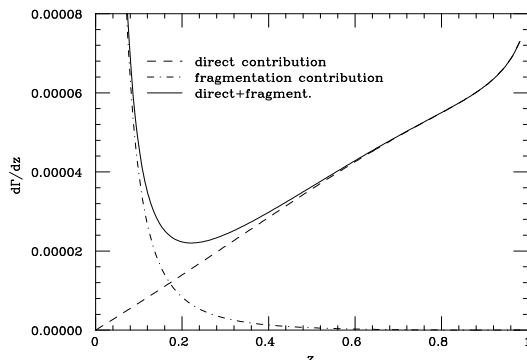


Figure 2: The leading-order photon spectrum $\Upsilon \rightarrow \gamma X$.⁶ Shown are the direct component (dashed curve), the fragmentation contribution (dot-dashed curve) and the sum (solid curve). Units $\langle \Upsilon | \mathcal{O}_1(^3S_1) | \Upsilon \rangle / m_b^2 = 6/\pi$ have been used, and $\alpha_s = 0.2$.

fragmentation contribution is important in the low- x_γ region, but suppressed with respect to the direct cross section for $x_\gamma \gtrsim 0.3$.^c

It is important to point out that the total decay width, including fragmentation processes, $\Gamma(\Upsilon \rightarrow \gamma X) = \int_0^1 dx_\gamma d\Gamma/dx_\gamma$, is not an infrared-safe observable. For $x_\gamma \rightarrow 0$, the fragmentation contribution rises like $1/x_\gamma$, characteristic of the soft bremsstrahlung spectrum, and cannot be integrated down to $x_\gamma = 0$. Therefore, in perturbation theory, only the photon energy spectrum can be calculated for $x_\gamma \neq 0$, after collinear singularities have been factorized into the fragmentation functions.

At present, only the region $x_\gamma \gtrsim 0.3$ is accessible to accurate measurement, because of the strong background from $\pi \rightarrow \gamma\gamma$ decays at low x_γ . A comparison between the theoretical prediction for the shape of the photon energy spectrum and the available experimental data can thus be restricted to the direct term. Such a comparison is shown in Fig. 3.⁷ The theoretical curve has been modified to account for the experimental efficiency and energy resolution and has been normalized to the data. The steep rise towards large

^cAt next-to-leading order, the fragmentation contribution involves also quark fragmentation, which is much harder than gluon fragmentation and may be important at larger values of x_γ .

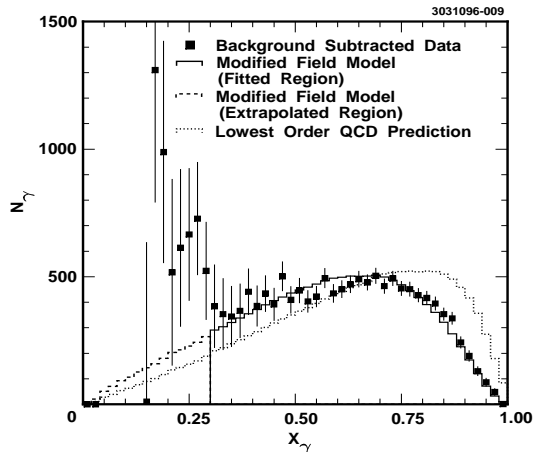


Figure 3: Experimental data for the photon spectrum compared to the efficiency-attenuated and energy-smeared prediction for the direct process at leading order.⁷

x_γ predicted by LO perturbative QCD is not supported by the experimental data. In particular for $x_\gamma \gtrsim 0.7$ the discrepancy is significant. Near the phase-space boundary $x_\gamma \rightarrow 1$, hadronization effects may become important, but they are not expected to modify the energy distribution significantly in the intermediate region $x_\gamma \sim 0.7$. To accommodate the data, different models have been proposed, including a parton shower Monte Carlo approach, also shown in Fig. 3, or parametrizations of non-perturbative effects in terms of an effective gluon mass.^{8,9} A proper theoretical understanding of the photon spectrum, however, requires a consistent inclusion of higher-order terms in the perturbative expansion as well as a careful examination of the contributions from relativistic corrections.

Given the above-mentioned theoretical problems, the present extractions of $\alpha_s(m_b)$ from the total radiative decay rate have to be considered uncertain. Even neglecting fragmentation contributions at low x_γ , model approaches have to be used to extrapolate the experimental data into the lower energy region to obtain the total decay rate, introducing uncontrolled theoretical uncertainties. Analysing the effect of higher-order corrections to the photon energy spectrum is thus not only an interesting problem of perturbative QCD per se, but will also lead to a more reliable determination of the strong coupling constant from radiative Υ decays.

3 Higher-order corrections to $\Upsilon \rightarrow \gamma + X$

The general factorization formula for the Υ decay rate of Eq. (1) is a double power-series expansion in the strong coupling α_s and the heavy-quark velocity v . Higher-order contributions to the LO estimate include both $\mathcal{O}(\alpha_s)$ corrections to the short-distance cross section and relativistic corrections $\mathcal{O}(v^2)$ due to the motion of the b quarks in the Υ bound state. Subsequently, we will discuss the different sources of higher-order corrections and examine their impact on the photon energy spectrum.

3.1 Soft-gluon resummation

It has been argued that potentially large logarithms $\ln(1 - x_\gamma)$ associated with the imperfect cancellation between real and virtual emission of soft gluons as $x_\gamma \rightarrow 1$ may contribute to all orders in perturbation theory.¹⁰ The resummation of these soft-gluon effects may then give rise to a Sudakov suppression $\sim 1/\exp(\alpha_s \ln^2(1 - x_\gamma))$ near the endpoint of the photon energy spectrum. However, a recent analysis reveals that the logarithms of $(1 - x_\gamma)$ cancel at each order in the perturbative expansion, in the non-relativistic limit where Υ decays proceed via the annihilation of a colour-singlet $b\bar{b}$ pair.¹¹ Accordingly, no Sudakov suppression arises at leading order in the velocity expansion. It has also been shown that the parton shower Monte Carlo approach,⁸ used in the recent determination of α_s from radiative Υ decays, does not correctly take into account gluon radiation in higher orders.¹¹

3.2 Relativistic corrections

Although the velocity of the b quarks in the Υ bound state is small, $v^2 \approx 0.1$, relativistic corrections may contribute significantly in certain regions of phase space. The NRQCD approach allows a systematic calculation of these corrections, introducing however additional non-perturbative matrix elements. Thus, in the absence of lattice results, one has to resort to phenomenological inputs to estimate the size of the higher-order terms in the velocity expansion.

The leading relativistic correction to the colour-singlet decay rate is of $\mathcal{O}(v^2)$ and can be expressed in terms of the ‘binding energy’ of the quarkonium $M_\Upsilon - 2m_b$.¹² Depending on the value of the b -quark pole mass, the $\mathcal{O}(v^2)$ correction may be as large as 25%, but it affects the shape of the photon spectrum only in the region $x_\gamma \gtrsim 0.8$.¹³ At $\mathcal{O}(v^4)$ in the velocity expansion, radiative Υ decays can proceed through the annihilation of colour-octet $n = {}^1S_0^{(8)}, {}^3P_{0,2}^{(8)}$ $b\bar{b}$ pairs into γg final states at $\mathcal{O}(\alpha_s)$. The enhancement of the colour-octet short-distance annihilation cross section $\sim \pi/\alpha_s$ partly compensates the strong

suppression of the long-distance matrix element $\sim v^4$. Since the LO short-distance process for colour-octet terms is kinematically restricted $\sim \delta(1 - x_\gamma)$, colour-octet contributions to the direct component of the decay spectrum are important only in the endpoint region $x_\gamma \gtrsim 1 - v^2 \approx 0.9$, taking into account an effective smearing of the delta function due to the resummation of higher-order terms in the velocity expansion.¹⁴ Colour-octet processes also contribute to radiative Υ decays by fragmentation.¹⁵ Both gluon and quark fragmentation turn out to be sizeable at low photon energies $x_\gamma \lesssim 0.3$. A separate measurement of the gluon fragmentation functions from the lower part of the energy spectrum, as suggested by the analysis of the colour-singlet decay channel, may thus not be feasible.

To summarize, relativistic corrections to the photon energy spectrum, although potentially sizeable, are concentrated in the upper and lower endpoint region and do not significantly modify the shape of the spectrum in the intermediate energy range $0.4 \lesssim x_\gamma \lesssim 0.9$.

3.3 Next-to-leading order QCD corrections

For intermediate photon energies, the spectrum is expected to be well described by the direct colour-singlet channel. The next-to-leading order corrections to the total decay rate of Eq. (2) are potentially large, depending on the choice of the renormalization scale. An earlier calculation yields

$$\Gamma_{\text{dir}}^{\text{NLO}}(\Upsilon \rightarrow \gamma X) = \Gamma_{\text{dir}}^{\text{LO}} \left(1 - \frac{\alpha_s(2m_b)}{\pi} (1.67 \pm 0.36) \right), \quad \mu = 2m_b, \quad (5)$$

with a large theoretical uncertainty of $\sim \pm 20\%$ coming from the numerical evaluation of the loop integrals.¹⁶

The calculation of the higher-order QCD corrections to the photon energy spectrum has been completed only recently.¹⁷ Generic diagrams that build up the decay rate in next-to-leading order are depicted in Fig. 4. Besides the usual self-energy and vertex corrections for photon and gluons (a), one encounters a large number of box diagrams (b) as well as gluon radiation off heavy quarks (c) and gluon splitting into gluons and light-quark-antiquark pairs (d). The evaluation of these amplitudes has been performed in the Feynman gauge and dimensional regularization has been used to calculate the singular parts of the amplitude. The masses of the $n_{\text{lf}} = 4$ light quarks have been neglected while the mass parameter of the b quark has been defined on-shell. The exchange of Coulombic gluons in diagram (4b) leads to a singularity $\sim \pi^2/2\beta_R$, which can be isolated by introducing a small relative quark velocity β_R . The Coulomb-singular part can be associated with the interquark potential of the

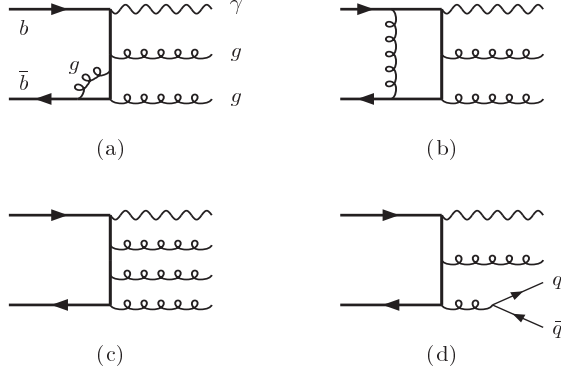


Figure 4: Generic next-to-leading order Feynman diagrams for direct radiative Υ decay.

bound state and has to be factored into the non-perturbative NRQCD matrix element. Only the exchange of transverse gluons contributes to the next-to-leading order expression for the short-distance annihilation rate. The infrared singularities cancel when the emission of soft and collinear final-state gluons and light quarks, described by universal splitting functions¹⁸, is added to the virtual corrections. The analytical result for the matrix element squared has been implemented in a Monte Carlo integration program, so that not only the inclusive decay rate but any distribution can be generated.

For the total width we obtain at next-to-leading order:

$$\Gamma_{\text{dir}}^{\text{NLO}}(\Upsilon \rightarrow \gamma X) = \Gamma_{\text{dir}}^{\text{LO}} \left[1 - \frac{\alpha_s(\mu)}{\pi} \left(1.71 + \beta_0(n_{\text{lf}}) \ln \frac{2m_b}{\mu} \right) \right], \quad (6)$$

where $\beta_0(n_{\text{lf}}) = (11N_C - 2n_{\text{lf}})/3$.¹⁷ The theoretical uncertainty due to the numerical phase-space integration has been reduced to $\lesssim 0.5\%$. The new, more accurate result for the total width is consistent with the previous calculation¹⁶ within the estimated numerical error.

The next-to-leading order corrections significantly flatten and deplete the photon energy spectrum for $x_\gamma \gtrsim 0.75$ as shown in Fig. 5, indicating that the discrepancy between theory and experiment at large photon energies can be reduced by the inclusion of higher-order QCD corrections. A more detailed discussion of the photon spectrum at NLO and a comparison with experimental data will be presented in a forthcoming publication.

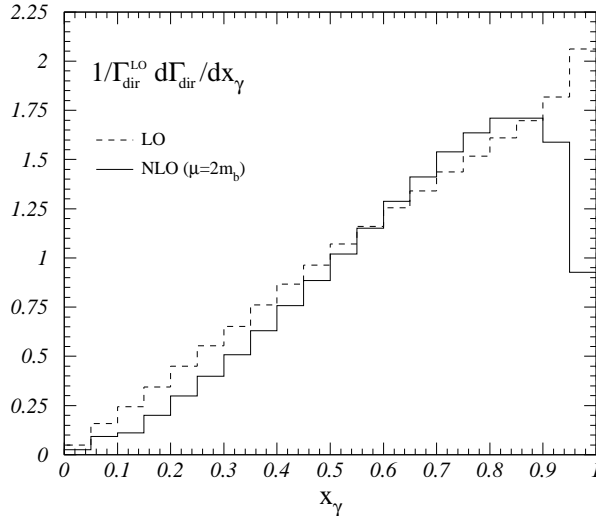


Figure 5: Photon energy spectrum for direct radiative Υ decays in leading and next-to-leading order ($\alpha_s = 0.2$).

4 Conclusions and outlook

The photon spectrum in radiative Υ decays is a very interesting laboratory to study perturbative and non-perturbative QCD effects. In the low- x_γ region, fragmentation contributions dominate, while near the upper endpoint of the spectrum relativistic corrections and hadronization effects become important. The next-to-leading order results presented here indicate that the intermediate region of the energy spectrum can be accounted for by perturbative QCD, once higher-order corrections are taken into account. This should allow a more reliable extraction of α_s from radiative Υ decays by restricting the analysis to the region of the energy spectrum that can be described by NLO perturbation theory.

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